BHP BILLITON — UNIVERSITY OF MELBOURNE SCHOOL MATHEMATICS COMPETITION, 2003: JUNIOR DIVISION

1. A bag contains a mixture of red and green marbles. If four red marbles are removed from the bag, then one-tenth of the remaining marbles are red. However, if four green marbles are removed instead of the four red, then one-fifth of the remaining marbles are red. How many red and green marbles were in the bag originally?

   Solution: Let r be the number of red marbles and g the number of green marbles in the bag originally. Then

   $$\begin{aligned} \frac{1}{10}(r+g-4) &= r-4 \\ \frac{1}{5}(r+g-4) &= r. \end{aligned}$$

   Therefore, equating expressions for $r+g-4$ gives

   $$\begin{aligned} 10r-40 &= 5r \\ \Rightarrow r &= 8. \end{aligned}$$

   So $r = 8$ and hence, $g = 36$.

2. In a party game ten people sit in a circle. Each person thinks of a number and tells it to their two neighbours sitting next to them in the circle. Then each person calculates and announces the average of the two numbers given to them by their neighbours. The diagram below shows the *average* numbers announced by each person. Find the number thought of by the person who announced the average number 2.

   Solution: Let x be the number thought of by the person who announced the average number 2. Then the person who announced the average of

4 must have thought of the number $6-x$ in order to produce the average number 3. Working around the circle in the clockwise direction in this manner shows that the person who announced 10 must have thought of the number $8+x$. Consequently

$$\begin{aligned}\frac{(8+x)+x}{2} &= 1 \\ \Rightarrow x &= -3.\end{aligned}$$

3. The surnames of four professional people are Bennett, Johnson, Ng and Wong, and their professions are accountant, lawyer, dentist and doctor (not necessarily in this order). The accountant and lawyer work in their offices, while the dentist and doctor work in their surgeries. The accountant receives free medical treatment and looks after Ng's accounts. Johnson does not know Bennett, although his surgery is in the same street as Bennett's office. Johnson and the accountant settle their accounts by mutual negotiation. Find the occupations of the four people.

   Solution: It is useful to use a matrix to solve this kind of problem. An initial reading gives the following information, where × denotes not applicable and ✓ denotes applicable.

	accountant	lawyer	doctor	dentist
Bennett				
Johnson	×	×	×	
Ng	×			
Wong				

   Further analysis produces the final matrix.

	accountant	lawyer	doctor	dentist
Bennett	×	✓	×	×
Johnson	×	×	×	✓
Ng	×	×	✓	×
Wong	✓	×	×	×

4. Five members of a computer club decided to buy a second-hand computer, sharing the cost equally. Later on, three new members joined the club and agreed to pay their share of the purchase price. If this

resulted in a saving of \$15 for each of the five original members, how much did the computer cost?

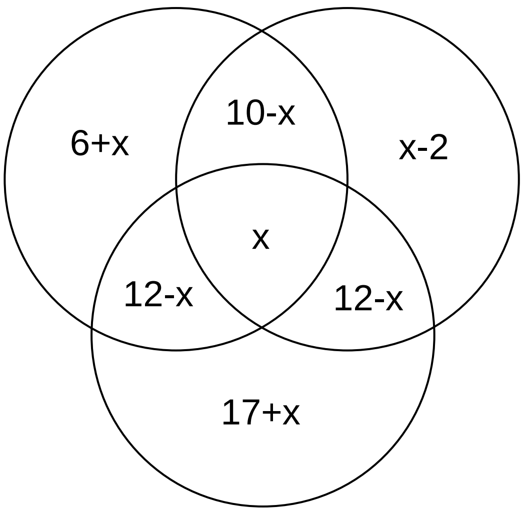
Solution: Suppose the computer cost $\$C$. Then

$$\begin{aligned} 8\left(\frac{C}{5} - 15\right) &= C \\ \Rightarrow \frac{8C}{5} - 120 &= C \\ \Rightarrow \frac{3C}{5} &= 120 \\ \Rightarrow C &= 200. \end{aligned}$$

Consequently, the computer cost \$200.

5. A survey was conducted of television programs viewed on a particular evening by 65 families. The channels involved were 2, 7, and 9. Of the 65 families, 28 had watched some channel 2 programs, 20 had watched some channel 7 programs, 41 had watched some channel 9 programs and 2 families had failed to watch any television that night. Some families had watched more than one channel: 10 had watched channels 2 and 7, 12 had watched channels 2 and 9, while 12 had watched channels 7 and 9. How many families had watched all three channels?

   Solution: This question is best solved using Venn diagrams. Let x be the number of families that watched all three channels. Then using the information given provides entries shown in the diagram.



Hence

$$\begin{aligned} 6 + x + 22 - x + x - 2 + 12 - x + 17 + x &= 65 - 2 \\ \Rightarrow x + 55 &= 63 \\ \Rightarrow x &= 8. \end{aligned}$$

6. A dog is two-thirds of the way across a railway bridge when it sees a train approaching, travelling at 30kph. The dog can just manage to get safely off the bridge by running (at the same constant speed) in either direction. How fast did the dog need to run, and how far is the train from the end of the bridge when the dog begins to run?

   Solution: Let the length of the bridge be l metres, and suppose that the train is d metres from the edge of the bridge when the dog begins to run. Suppose that the dog runs at v kph. This gives us the following two equations

   $$\frac{2}{3}\frac{l}{v} = \frac{d+l}{30}$$

   and

   $$\frac{1}{3}\frac{l}{v} = \frac{d}{30}.$$

   These two equations give $2 = \frac{d+l}{d}$ so that $d = l$ and hence, $v = 10$. Hence, the dog needs to run at 10 kph, and the train is the length of the bridge away when the dog begins to run.

7. A teacher asks a member of the class to think of a three-digit number abc where a, b and c are natural numbers (1, 2, 3, 4, 5, 6, 7, 8 or 9). The teacher then asks the student to write down the five numbers acb, bac, bca, cab and cba, add them together, and announce the total T. When told the value of T the teacher was able to determine the original number abc. In the case when $T = 3194$, can you find the number abc?

   Solution: Write the five numbers acb, bac, bca, cab and cba in the form

   $$\begin{gathered} 100a + 10c + b \\ 100b + 10a + c \\ 100b + 10c + a \\ 100c + 10a + b \\ 100c + 10b + c. \end{gathered}$$

The total T of these five numbers can be written

$$
\begin{aligned}
T &= 200(a+b+c) + 20(a+b+c) + 2(a+b+c) \\
&\quad -(100a + 10b + c) \\
T &= 222(a+b+c) - abc \\
T + abc &= 222(a+b+c).
\end{aligned}
$$

Hence, we require a number $k = a + b + c$ such that

$$222k = 3194 + abc.$$

Now $222k$ lies between 3194 and $3194 + 1000$, which limits k to the possible values 15, 16, 17 or 18. If $k = 16$, then $222k - 3194 = 358$, and $3 + 5 + 8 = 16$.