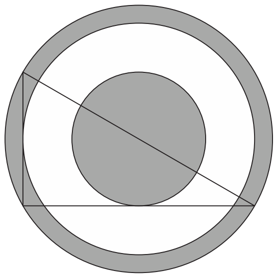
1. Kim has just completed a 21 kilometre bike trip. If she had been able to ride 2 kilometres per hour faster, she would have completed her trip 15 minutes earlier. Find her speed.

2. Five letters are written down from left to right satisfying

   (a) only the letters A, C, G and T are used; and
   (b) no neighbouring letters are the same.

   What is the probability that "CAT" will appear as three consecutive letters written from left to right?

3. In the following diagram, each of the three circles is centred at the midpoint of the longest side of the triangle. Prove that the two shaded regions have the same area.



4. In the 18th century, Euler proved the remarkable fact that

$$\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\frac{1}{5^2}+\cdots=\frac{\pi^2}{6}.$$

   Use this to determine the value of

$$\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}+\frac{1}{9^2}+\cdots.$$

5. Find all positive integers less than 10,000 which are equal to five times the product of their digits.

6. Recall that the absolute value of x is defined as

$$|x|=\begin{cases} x, & \text{if } x\geq 0 \\ -x, & \text{if } x<0 \end{cases}.$$

   Find all real values of x that satisfy

$$|x|-|x+2|+|x+4|-|x+6|+|x+8|=|x+1|-|x+3|+|x+5|-|x+7|+|x+9|.$$

7. Express the number $\frac{1}{2006}$ as the sum of the reciprocals of distinct integers of size less than 2006 — that is,

$$\frac{1}{2006}=\frac{1}{a_1}+\frac{1}{a_2}+\cdots+\frac{1}{a_n},$$

   where $-2006<a_i<2006$ for all $i=1,2,\ldots,n$.