1. Find all positive integer solutions of the equation $4m + 3n = 200$.

2. Two hospitals in the same town treat the same number of patients each month. The first hospital cures more patients each month than does the second. Each patient suffers from one of two possible diseases. It turns out that the second hospital cures a higher percentage of patients with disease 1, and also a higher percentage of patients with disease 2. Is this possible? (For your answer, you should either provide a numerical example consistent with the above observations, or prove that it is not possible).

3. Consider the sequence $1, 5, 6, 25, 26, 30, 31, \ldots$ consisting of all positive integers which are either powers of 5, or sums of distinct powers of 5. (For example, $31 = 5^2 + 5^1 + 5^0$). Find the $50^{th}$ term in this sequence.

4. Consider a (mythical) chess-piece that may only move in the $+x$ (i.e. to the right) and $+y$ (i.e. up) direction. Assume that this chess-piece is initially positioned at the bottom, left-hand corner of the $N \times N$ chessboard. (a) Show that the number of different paths from the initial position (that is, the bottom-left square) to the top right square is $(2N)^{2N} - 1$. (b) Using the answer to the previous part, or otherwise, find the total number of squares below all such paths. (For example, if $N = 2$, the answer is 1. If $N = 3$, the answer is 12, as of the six possible paths, there are 4 squares below one path ($yyxx$), three squares below one path ($yxyx$), two squares below each of two paths ($yxx$ and $xyyx$), one square below one path ($xyxy$), and no squares below one path ($xxyy$).

5. You are given four points, lying in the plane, and told that the distance between any pair of points is not less than $\sqrt{2}$ and not greater than 2. Prove that these points must lie at the four vertices of a square.

6. At a picnic, 15 friends decide to play rugby 7s. One of them is chosen to be the referee, and the others are split into two teams, each of 7 members. For balance, they decide that the two teams should have the same total weight. (Assume the weight of each player is an integer number of kilograms). They find that no matter who is chosen as the referee, this can always be done. Prove that all 15 friends must have the same weight.

7. Let $x_1, x_2, \ldots, x_n$ be positive real numbers, such that

$$\sum_{i=1}^{n} \frac{1}{n + x_i - 1} > 1.$$ 

Prove that $x_1x_2\ldots x_n < 1$. 
