These questions are designed to test your ability to analyse a problem and to express yourself clearly and accurately.

The following suggestions are made for your guidance.

1. Considerable weight will be attached by the examiners to the method of presentation of a solution. Candidates should state as clearly as they can the reasoning by which they arrived at their results. In addition, more credit will be given for an elegant than for a clumsy solution.

2. The seven questions are not of equal length or difficulty. Generally, the later questions are more difficult than the earlier questions.

3. It may be necessary to spend considerable time on a problem before any real progress is made.

4. You may need to do considerable rough work but you should then write out your final solution neatly, stating your arguments carefully.

5. Credit will be given for partial solutions; however a good answer to one question will normally gain you more credit than sketchy attempts at several questions.

Textbooks are NOT allowed. Electronic calculators, tables, etc., may be used. Computers may not be used. Calculators capable of storing text should have their memories erased before use. Otherwise normal examination conditions apply.

Candidates may attempt all questions.

Warning: Make sure you have the correct problems (Senior, Intermediate or Junior) in front of you.
1. Consider an \(a \times b\) rectangle, as shown below, \(a > b\). Clearly its area is \(ab\). Now fold it on a diagonal, giving a five-sided figure, as in the diagram below. What is the area of the five-sided figure?

![Diagram of a rectangle and a five-sided figure](image)

2. In a river, there is an H-shaped bridge, with 5 arms labelled \(a, b, c, d, e\). A sailing ship can’t sail upstream under the bridge, as its mast is too tall. An earthquake strikes, and each of the 5 arms of the bridge has, independently, probability \(1/2\) of collapsing, thus allowing the ship to sail through the spot previously blocked by that arm. What is the probability that the ship can sail upstream after the earthquake? (You may assume that the earthquake did not damage the ship).

![Diagram of an H-shaped bridge](image)

3. Alan, Betty, Chris and Don play table tennis. Two of them play a game, and the loser leaves the table, to be replaced by one of the the other two players who have sat out the game. The person who has sat out for the greatest number of consecutive games comes to the table for the next game. (If they have sat out for an equal number of games, either can come to the table). At the end of the day, Alan played 61 games, Betty played 22 games, Chris played 21 games and Don played 20 games. Who played in the 33rd game?

4. An odd number of people are standing in a field. The distance between each pair of people is distinct (i.e. different). They are each armed with a water pistol, and at the same precise moment each person fires at (and hits) the person nearest to them. Prove that at least one person does not get wet.
5. All vertices of a polygon $P$ lie at points with integer co-ordinates in the plane (that is to say, both their co-ordinates are integers), and all sides of $P$ have integer lengths. Prove that the perimeter of $P$ must be even.

6. It is asserted that one can find a subset $S$ of the nonnegative integers such that every nonnegative integer can be written uniquely in the form $x + 2y$ for $x, y \in S$. Prove or disprove the assertion.

7. Consider a game in which you have $n > 1$ identical coins. They are to be placed in a row. The first coin is placed down, and the second is then placed to the right of the first, the third is placed immediately to the right of the second, and so on. The rules of the game are that you can move a coin only when there is another coin immediately to the left of the coin you wish to move, and you can only place a coin down if there is a coin immediately to the left of the coin you are placing down. The aim of the game is to move a coin as far to the right as possible. With two coins you can’t move either of them. With three coins (as shown in the left-hand figure below), you can pick up the coin in position 2 and place it in position 4. You can’t move any coins further to the right. With $n > 3$ coins, what is the rightmost position to which you can move a coin, given a starting configuration in which the coins are placed in positions 1, 2, 3, ..., $n$?

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1 & 2 & 3 & 4 \\
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1 & 2 & 3 & 4
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