The University of Melbourne–Department of Mathematics and Statistics
School Mathematics Competition, 2014

INTERMEDIATE DIVISION

Time allowed: Three hours

These questions are designed to test your ability to analyse a problem and to express yourself clearly and accurately. The following suggestions are made for your guidance:

(1) *The examiners will attach considerable weight to the method whereby a solution is presented. Candidates should state as clearly as they can the reasoning by which they arrived at their results. In addition, more credit will be given for an elegant than for a clumsy solution.*

(2) *The **seven** questions are not of equal length or difficulty. Generally, the later questions are more difficult than the earlier questions.*

(3) *It may be necessary to spend considerable time on a problem before any real progress is made.*

(4) *While you may need to do considerable rough work, you should write your final solution neatly, stating your arguments carefully.*

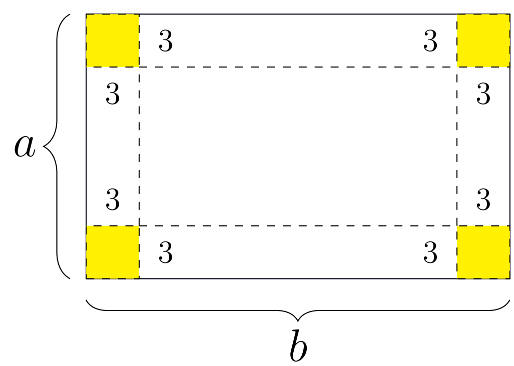
(5) *Credit will be given for partial solutions; however a good answer to one question will normally gain you more credit than sketchy attempts at several questions.*

*Textbooks, electronic calculators and computers are **NOT** allowed. Otherwise normal examination conditions apply.*

(1) Using each of the digits 1,2,3,4,5 exactly once, write the product of a 1-digit number times a 2-digit number giving a 2-digit number.

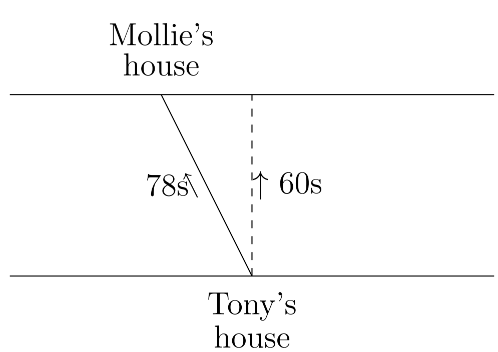
(2) Albert does not remember his own phone number, however he does remember that it has 8 digits, it begins with a 9, ends with a 2, and it is equal to the product of three consecutive even numbers. What is the second last digit of Albert's phone number?

(3) Make a box from a rectangle of size $a \times b$ by cutting out four squares and folding up the edges. If you cut out squares of size 3×3 as in the diagram, the volume of the box is the same as the volume of the box made from cutting out squares of size 5×5. What is the volume of the box if you cut out squares of size 8×8?



(4) A jar contains coins made up of 5 cent, 10 cent, 20 cent, 50 cent, 1 dollar and 2 dollar coins. The average value of a coin in the jar is 82 cents. If six 5 cent coins are added to the jar then the average value of a coin in the jar drops to 40 cents. How many 20 cent coins are in the jar?

(5) Tony often swims across the river from his house to Mollie's house. If there is no current then Tony can swim directly to Mollie's house in 78 seconds. If Tony instead swims directly across the river in no current it takes him 60 seconds. If there is a current moving at half the speed that Tony can swim then how long does it take Tony to swim directly to Mollie's house and back?



(6) Prove that one can find 4027 consecutive positive integers such that the sum of the squares of the first 2014 integers is equal to the sum of the squares of the last 2013 integers.

(7) Write the numbers 1 to 2014^2 in a 2014×2014 grid with 1 to 2014 in increasing order from left to right in the top row, 2015 to 4028 in increasing order from left to right in the second row and so on.

1	2	3		2013	2014
2015	2016				4028
					2014^2

Shade half the squares of the grid any way you like, with the restriction that each row and each column has the same number of shaded and unshaded regions. A chessboard is an example. Prove that the sum of the numbers in the unshaded regions equals the sum of the numbers in the shaded regions no matter how you shaded the squares.