The University of Melbourne–Department of Mathematics and Statistics
School Mathematics Competition, 2014
JUNIOR DIVISION
Solutions

1. Old boots and used socks. The jug contains 250ml of Grange and 1000ml of lemonade. Hence four fifths of the mixture is lemonade ($1250 \times (4/5) = 1000$) and one fifth is wine ($1250 \times (1/5) = 250$). Therefore, of the 250ml poured back into the bottle of Grange, $250 \times (4/5)$ml $= 200$ml is lemonade and $250 \times (1/5)$ml $= 50$ml is wine. Before drinking, the bottle of Grange thus contains 550ml of wine and 200ml of lemonade.

2. Springfield University – School Mathematics Competition. To award the top 30 students a different whole dollar amount Homer needs at least

$$1 + 2 + 3 + \cdots + 29 + 30 = (1 + 30) + (2 + 29) + \cdots + (15 + 16) = 15 \times 31 = 465$$

dollars.

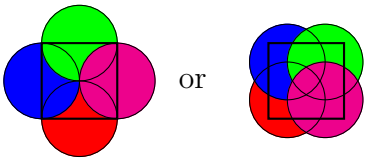
3. Wet and wild. Since each tap can be open or closed there are $2 \times 2 \times 2 \times 2 = 16$ possible combinations. Marge stays dry if

(1) All four taps are closed. There is one way to do this.
(2) Three taps are closed and one is open. There are four ways to do this.
(3) The two right-hand taps are closed and the two left-hand taps open.
(4) The two left-hand taps are closed and the two right-hand tabs are open.

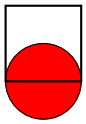
If three or more taps are open Marge will definitely get wet.
The total number of combinations to stay dry is thus $1 + 4 + 1 + 1 = 7$ out of 16, so that the total number of combinations for Marge to get wet is therefore $16 - 7 = 9$. The probability that she will get wet is $9/16$. Equivalently, she has a $9/16 \times 100 = 56.25\%$ chance to get wet.

4. Thanassis. Since Euclid wins a total of 4 books and Pythagoras loses a total of 5, their combined tally stands at a loss of 1 book. Since no books were burned, this means that Archimedes' total winnings were 1 book. Since he won 4 games of Thanassis, earning him 8 books, he must have lost 7 games. This makes for a total of 11 games played.

5. Circling the square. You can certainly cover the table with four tablecloths, and



are just two examples of how Peta Credlin might have done it.
So could she have managed with three?
Well, let's try. Our table has four corners which need to be covered. If we have three tablecloths, then at least one of these must cover two corners. Up to rotation, there is only one way to do this:



But we are now left with three uncovered edges of the table. Since one tablecloth can at most cover one full edge, we cannot succeed.

6. Friends. Call a student *Even* if they have an even number of friends, and call a student *Odd* if they have an odd number of friends.
To count the number of friendships at your school you can go round and ask everyone (including yourself!) how many friends they have. If you add up all these numbers, you have double-counted the number of friendships, because if Jack and Jill are friends, Jack will have included Jill in his count and Jill will have included Jack in hers. Hence you need to divide your total tally by 2. In other words, *before* you do this division you must have had an even number. But in your tour around the school Even students have given you an even number and Odd students have given you an odd number. Because an odd number of odd numbers adds up to an odd number (odd+odd=even, odd+odd+odd=odd, odd+odd+odd+odd=even, etc.) there must have been an even number of Odd students.

We must now show that you receive an even number of emails. You receive an email from your Even friends (EFs) and Odd non-friends (ONFs), and you do not receive an email from your Odd friends (OFs) and Even non-friends (ENFs). So the question is: why are there an even number of EFs and ONFs combined?
We have already shown that at *any* school, there must be an even number of Odd students, and so the question is: Can we create a scenario where your EFs and ONFs precisely make up the Odd students at a school? The answer is "yes"; simply get yourself expelled! Then your EFs (losing you as their school friend) become Odd, your ONFs (not losing a friend) stay Odd, your OFs (losing you) become Even and your ENFs (again not losing a friend) stay Even. In conclusion, the EFs together with the ONFs make up the Odd students at your school once you have been expelled. Hence their combined number must be even.