The University of Melbourne–Department of Mathematics and Statistics
School Mathematics Competition, 2015

JUNIOR DIVISION

Time allowed: Two hours

These questions are designed to test your ability to analyse a problem and to express yourself clearly and accurately. The following suggestions are made for your guidance:

(1) *Considerable weight will be attached by the examiners to the method of presentation of a solution. Candidates should state as clearly as they can the reasoning by which they arrived at their results. In addition, more credit will be given for an elegant than for a clumsy solution.*

(2) *The **six** questions are not of equal length or difficulty. Generally, the later questions are more difficult than the earlier questions.*

(3) *It may be necessary to spend considerable time on a problem before any real progress is made.*

(4) *You may need to do considerable rough work but you should then write out your final solution neatly, stating your arguments carefully.*

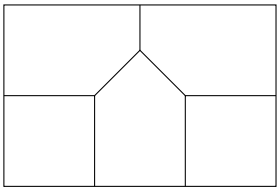
(5) *Credit will be given for partial solutions; however a good answer to one question will normally gain you more credit than sketchy attempts at several questions.*

*Textbooks, electronic calculators and computers are **NOT** allowed. Otherwise normal examination conditions apply.*

1. Year of the sheep. On New Years Eve you and your family play a game of Bingo with 2015 Bingo cards numbered 1 to 2015. Your Dad is drawing one card at a time and calling out the number on that card. You are of course much too clever for such a boring game and cannot wait for the clock to strike midnight and 2016 to kick off. To break the tedium you start thinking about 2016 and wonder what is the minimum number of cards that your Dad must draw to be guaranteed he has called out two numbers whose sum makes 2016.
What is this number of cards?

2. The fast and the furious. To put an end to any further leadership speculations, Tony Abbott, Malcolm Turnbull and Julie Bishop decide to race each other at the annual Pollie Pedal, with the winner being awarded the Prime Ministership. Just after the start of the race Malcolm is in the lead, followed by Tony, and then Julie. During the remainder of the race Malcolm and Tony pass each other 18 times, Tony and Julie pass each other 23 times, and Malcolm and Julie pass each other 13 times.
Who finishes first and takes home the leadership of the country?

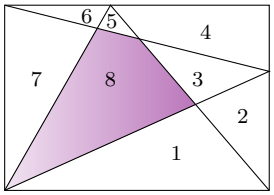
3. Not for the faint hearted. On its website the Melbourne Gaol warns prospective visitors that due to an "unexplained presence" night time tours are not recommended for children under 12 years of age. Indeed, the Gaol is a scary place where Ned Kelly's ghost roams free. Every night he tries to frighten visitors (hopefully all of the required age) by doing his own tour of the building, crossing through each of the Gaol's sixteen different walls exactly once. Ned can choose any of the five rooms of the Gaol as the starting point for his ghost tour, or even start outside the building, and does not necessarily have to finish where he started.



Will it ever be possible for him to cross through every wall exactly once? Clearly explain your answer.

4. Zingers. To put an end to any further leadership rumblings, Bill Shorten and Tanya Plibersek decide to play a game of Zingers. In Zingers the numbers 1 to 21 are written on twenty-one pieces of paper and put face up on a table. The players take turns to remove two pieces of paper of their choice and replace these with one new piece of paper on which they write the difference of the two numbers just removed. For example, if a player chooses to remove the two numbers 19 and 8 they should replace these by a new piece of paper on which they write 11, or if they choose 13 and 13 (after a while numbers can occur more than once) they should replace these by a new piece on which they write 0. Bill wins if the last remaining number is even and Tanya if the last number is odd. (The number 0 is even.)
Who of the two wins the leadership of the opposition?

5. Polycrates & Pythagoras. Polycrates ruled the Greek island of Samos from 538 BC until 522 BC. When he turned 70, Polycrates thought it was time for his son Trigon and daughter Plateia to take over part of the island's rule. He took a map of Samos, which happens to be a perfectly rectangular island, and divided it into eight counties as shown on the right. He promised Plateia the county numbered 8, but then worried which counties he should hand over to his son so that both Trigon and Plateia would rule over an equally large area. After consulting with Professor Pythagoras from The University of Melbourne the issue was quickly resolved. Which counties did Professor Pythagoras tell Polycrates to hand over to Trigon?



6. Kaapa Tjampitjinpa. Kaapa Tjampitjinpa (1926–1989) is probably Australia's most widely-known exponent of the famous Papunya School of dot painting. In his autobiography Tjampitjinpa noted that if every point on a straight line (remember that a straight line has infinitely many points) is coloured by either a blue or a red dot (Tjampitjinpa used an infinitely thin brush to make infinitely small dots) then there will always be three equally-spaced points on that line that have the same colour.
Explain Tjampitjinpa's observation in a mathematically precise way.